7.1/7.4 Introduction to Matrices and Determinants

7.1 –Introduction to Matrices

A ***matrix*** is a rectangular array of numbers. Each number in a matrix is called an ***element*** of the matrix.

The *order or dimension* of a matrix is determined by the number of rows by the number of columns (rows x columns).

$\begin{bmatrix} 7 & -2 & 8 & 5 \\ 4 & -1 & 0 & 2 \\ -2 & 3 & 5 & 12 \end{bmatrix}$ is a 3 x 4 matrix – 3 rows and 4 columns

(**Rows** – left to right, **Columns** – top to bottom)

Matrices can be used to rewrite a system of equations. ***Replace missing variables with 0.***

Using the system of equations: $\begin{cases} 2x - 3y + z = 2 \\ x + 0\text{y} - 3z = 4 \\ 4x - y + 4z = 3 \end{cases}$ we can create a matrix.

The *augmented matrix* The *coefficient matrix* The *constant matrix*

Ex: Create the augmented matrix, the coefficient matrix, and the constant matrix for the following system of equations:

$$\begin{cases} \quad 2x - 3y \ = 4 \\ x \ + 2y - 3z = 4 \\ 4x - y + 2z = 6 \end{cases}$$

The *augmented matrix* The *coefficient matrix* The *constant matrix*

7.4 – Determinants

Determinant of a 2x2 Matrix

<u>Each square matrix has a value called the determinant.</u>

The *determinant* of a 2x2 matrix A is: $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb.$

The determinant can also be identified as: det(A).

Examples:

1. Find the determinant of A = $\begin{bmatrix} 9 & 1 \\ 8 & 2 \end{bmatrix}$

2. Find the determinant of A = $\begin{bmatrix} 3 & -4 \\ 2 & 5 \end{bmatrix}$

3. Find the determinant of A = $\begin{bmatrix} 2 & -1 \\ 8 & -7 \end{bmatrix}$

4. Find the determinant of A = $\begin{bmatrix} -2 & -1 \\ -4 & -3 \end{bmatrix}$

<u>Can be used to solve a 2 equation / 2 variable linear system.</u>

Determinant of a 3x3 Matrix

<u>Evaluate a Determinant Using the Shortcut Method (column extension)</u>

Extend the first 2 columns and multiply along the 3 diagonals going down and the 3 diagonals going up

$$|A| = det(A) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix} \rightarrow \left| \begin{array}{ccc|cc} a & b & c & a & b \\ d & e & f & d & e \\ g & h & k & g & h \end{array} \right.$$

$det(A)$ = sum of down diagonals – sum of up diagonals = (aek)+(bfg)+(cdh) – (gec)-(hfa)-(kdb)

Using *column extension* evaluate the determinant of $\begin{vmatrix} 5 & -3 & 2 \\ 2 & 1 & 4 \\ 0 & -2 & -1 \end{vmatrix}$

Can be used to solve a 3 equation / 3 variable linear system.

A matrix has a determinant of 0 if:

1. A row (column) consists entirely of zeros.
2. Two rows (columns) are identical.
3. One row (column) is a constant multiple of a second row (column).